Propagation of Light in Doubly Special Relativity

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(Received January 5 2004)

Abstract

In an attempt to clarify what is the velocity of a particle in doubly special relativity, we solve Maxwell's equations invariant under the position-space nonlinear Lorentz transformation proposed by Kimberly, Magueijo, and Medeiros. We show that only the amplitude of the Maxwell wave, not the phase, is affected by the nonlinearity of the transformation. Thus, although the Maxwell wave appears to have infinitely large energy near the Planck time, the wave velocity is the same as the conventional light velocity. Surprisingly, the velocity of the Maxwell wave is not the same as the maximum signal velocity determined by the null geodesic condition, which is infinitely large near the Planck time and monotonically decreases in time to the conventional light velocity when time approaches infinity. This implies that, depending on the position of the particle in question, the light cone determined by Maxwell's equations may be inside or outside the null cone determined by the null geodesic equation, which may lead to the causality problem.

PACS numbers: 03.30.+p, 11.30.Cp, 04.60.-m, 04.50.+h.

Keywords: Doubly Special Relativity, Nonlinear Transformation, Maxwell's Equations

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I. INTRODUCTION

Doubly special relativity (DSR), which is a generalization of special relativity with two observer-independent scales, has attracted much attention recently in an attempt to describe the phenomenological aspects of physics on the Planck scale. It was first introduced as a generalization of the Lorentz transformation, respecting the principle of relativity in a linear fractional form [1-3]. It was recently suggested again as a generalization of special relativity to include one more invariant scale, in addition to the speed of light, introduced in a modified Lorentz transformation in momentum space[4], and it was subsequently shown to be a nonlinear representation of the Lorentz group [5,6]. It has also been pointed out that these theories can be understood as particular bases of the κ-Poincare theory based on quantum (Hopf) algebra, and the resulting energy-momentum space admits description in terms of a deSitter-type geometry[7]. Although DSR defined in momentum space has provided a possibility of explaining phenomena such as ultra-high-energy cosmic rays with energies above the Greisen-Zatsepin-Kuzmin (GZK) cut-off and high-energy gamma rays [4,5,8] and a candidate for a varying light velocity theory for cosmology[8], some questions on the interpretation and the consistency of the proposal have been raised.

First of all, it is not clear how to define the velocity of a particle unambiguously in an inertial frame[9]. The authors of Ref. 10 has also raised the possibility of large-scale nonlocality in the DSR defined in momentum space. The root of such problems lies in the fact that we do not know the exact position-space transformation corresponding to the modified Lorentz transformation defined in momentum space, such as those suggested in Refs. 4 and 5. In position space, the velocity of a particle is unambiguously defined as dx^i/dt and the unique momentum of the particle from the velocity. Also, the (non)locality of field theory can be addressed unambiguously in position space whereas in Ref. 10, a Fourier transformation was used to obtain the behavior of the field in position space. To use a Fourier transformation, one has to rely on the quantum-mechanical relation $p_i = -i\partial/\partial x^i$ for discussion of classical theory. Although there have been attempts to obtain the position-space transformation corresponding to the given momentum-space nonlinear Lorentz transformations [11,12], the results depend on the momentum, and their physical interpretation is not yet clear.

Thus, if the physical implications of nonlinear relativity are to be understood, it is preferable to start with the modified Lorentz transformation defined in position space. The nonlin-

ear representation of the Lorentz group in position space with a large invariant length scale was first proposed by Fock[1], the so-called Fock-Lorentz transformation, and elaborated by Manida[2] and Stepanov[3]. Recently, Kimberly, Magueijo, and Medeiros (KMM) proposed a nonlinear relativity in position space with a Planck scale invariant[11], which is relevant in applications to Planck-scale physics. The purpose of this paper is to study the physical implications of nonlinear relativity theories defined in position space.

In the next section, we study the space-time structure of the position-space nonlinear relativity proposed by KMM. We show that the maximum signal velocity determined by the null geodesic condition is infinitely large near the Planck time and monotonically decreases in time to the light velocity, c, when time becomes infinite. In Section III, we solve Maxwell's equations invariant under the KMM transformation proposed in Ref. 11. Although the Maxwell wave appears to have infinitely large energy near the Planck time, its velocity is shown to be the same as the conventional light velocity. Finally, in Section IV, we discuss some implications of our results.

II. SPACE-TIME STRUCTURE OF NONLINEAR RELATIVITY IN POSITION SPACE

In this section, we study the space-time structure of the nonlinear relativity proposed by KMM[11] and will briefly discuss that of the Fock-Lorentz case [1-3]. The nonlinear realization of the Lorentz group may be obtained by means of a transformation V such that the new boost generators of the Lorentz group with respect to space-time coordinates are

$$K^i = V^{-1} {L_0}^i \; V, \tag{1}$$

where

$$L_{\alpha\beta} = x_\alpha \frac{\partial}{\partial x^\beta} - x_\beta \frac{\partial}{\partial x^\alpha} \tag{2}$$

are the standard Lorentz generators. Despite this transformation, the new generators satisfy the ordinary Lorentz algebra,

$$[J^i, K^j] = \epsilon^{ijk} K_k, \quad [K^i, K^j] = \epsilon^{ijk} J_k, \quad [J^i, J^j] = \epsilon^{ijk} J_k, \tag{3}$$

where $J^i = \epsilon^{ijk} L_{jk}$. Thus, the new generators constitute a new (nonlinear) representation of the Lorentz algebra. KMM chose V as

$$V = e^{\frac{t_P}{t} D}, \tag{4}$$

where $D = x^a \partial_a$ and t_P represents a new invariant scale, the Planck time. Now, due to Eq. (1), the boost generators for nonlinear relativity take the form

$$K^i = L_0{}^i - \frac{t_P x^i}{t^2} D, \tag{5}$$

which in turn induces a nonlinear representation of a Lorentz transformation. The nonlinear Lorentz transformation is given by $x'^\mu = V^{-1} e^{\omega^{\alpha\beta} L_{\alpha\beta}} V x^\mu$:

$$\begin{aligned} t' &= \gamma(t - vx^3)(1 - \frac{t_P}{t}) + t_P, & (6) \\ x'^3 &= \gamma(x^3 - vt)[(1 - \frac{t_P}{t}) + \frac{t_P}{\gamma(t - vx^3)}], & (7) \\ x'^a &= x^a[(1 - \frac{t_P}{t}) + \frac{t_P}{\gamma(t - vx^3)}], \quad a = 1, 2, & (8) \end{aligned}$$

where $v = \tanh \omega^{03}$ is the relative velocity between two frames, which is assumed to be in the x^3-direction, and $\gamma = \frac{1}{\sqrt{1-v^2}}$ is the Lorentz factor. Note that the transformations in Eqs. (6)-(8) are singular at $t = t_P$, the Planck time. Thus, the theory describes the region of space-time after or before the Planck time. We also note that changing the sign of t_P (or t) in Eq. (4) leads to another nonlinear representation of the Lorentz group, so one can implement the time reversal symmetry in the system, as noted by KMM [11]. As KMM pointed out [11], the nonlinearity of the representation destroys translational invariance. Thus, the addition law for the coordinate variables is not obvious and needs further study, as is the case for the momentum variables in DSR defined in momentum space. Even though the position space DSR proposed by KMM and by FR nonlinear relativity poses many problems, the position space DSR can be useful in understanding how to define the velocity of light. To this end, we will consider the null geodesic condition in this section and the Maxwell equation in the next section.

The nonlinear KMM transformation, Eqs. (6)-(8), can be represented as a linear Lorentz transformation in terms of the new variables

$$\tilde{x}^\mu = x^\mu (1 - \frac{t_P}{t}). \tag{9}$$

It is easy to check that the quantity

$$\tilde{t}^2 - \vec{\tilde{x}} \cdot \vec{\tilde{x}} = (t^2 - \vec{x} \cdot \vec{x})(1 - \frac{t_P}{t})^2 \tag{10}$$

is invariant under the KMM transformation. The metric of the coordinate system is then given by

$$ds^2 = d\tilde{t}^2 - d\vec{\tilde{x}} \cdot d\vec{\tilde{x}} = dt^2 - [\frac{t_P}{t^2}\vec{x}dt + (1 - \frac{t_P}{t})d\vec{x}\,]^2. \tag{11}$$

The explicit components of the metric tensor read

$$\begin{aligned} g_{00}^{KMM} &= 1 - \frac{t_P^2\ \vec{x} \cdot \vec{x}}{t^4}, &(12)\\ g_{0i}^{KMM} &= -\frac{t_P x^i}{t^2}(1 - \frac{t_P}{t}), &(13)\\ g_{ij}^{KMM} &= -\delta_{ij}(1 - \frac{t_P}{t})^2, &(14) \end{aligned}$$

with its determinant given by

$$g^{KMM} = -(1 - \frac{t_P}{t})^6. \tag{15}$$

The maximum velocity of the signal propagation in DSR is determined by the condition for the null geodesic, $ds^2 = 0$, in Eq. (11):

$$[\vec{v} + (\frac{1}{1 - \frac{t_P}{t}})\frac{t_P \vec{x}}{t^2}]^2 = \frac{1}{(1 - \frac{t_P}{t})^2}. \tag{16}$$

Thus, in the direction where $\vec{x}$ is parallel to $\vec{v}$, the maximum *signal velocity* is given by

$$v = \frac{1}{1 - \frac{t_P}{t}}(1 - \frac{t_P x}{t^2}). \tag{17}$$

This shows that the maximum signal velocity depends on the position in space-time. The signal velocity at the origin, $\vec{x} = 0$, is infinitely large near the Planck time and monotonically decreases in time to the conventional light velocity when time approaches infinity. This is exactly what one expected when DSR was first introduced for application to Planck-scale physics [4,5,8].

To diagonalize the metric in Eq. (11), we define a new radial coordinate r as

$$r = \sqrt{\vec{x} \cdot \vec{x}}\ t(t - t_P), \tag{18}$$

which turns the metric in Eq. (11) into

$$ds^2 = \frac{1}{t^4}[(\sqrt{t^4 - \frac{4r^2}{t^2}}\; dt + \frac{2r}{t\sqrt{t^4 - \frac{4r^2}{t^2}}}\; dr)^2 - \frac{t^6}{t^6 - 4r^2}\; dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \quad (19)$$

If new variables T and $\tilde{R}$ as

$$T = t^2, \quad \tilde{R} = \frac{2r^2}{T^3} \quad (20)$$

are introduced, the metric in Eq. (19) may be put into a diagonal form:

$$\begin{aligned} ds^2 &= \frac{1}{T^2}[\frac{Td\tau^2}{1-2\tilde{R}} - \frac{dr^2}{1-2\tilde{R}} - r^2(d\theta^2 + \sin^2\theta d\phi^2)] && (21) \\ &= \frac{1}{T(1-2\tilde{R})}[d\tau^2 - \frac{1-2\tilde{R}}{T}dl^2], && (22) \end{aligned}$$

where the new time variable τ is defined by

$$\tau = \frac{1}{2}(T + \tilde{R}T), \quad (23)$$

and the metric of the space-like hypersection is given by

$$dl^2 = \frac{dr^2}{1-2\tilde{R}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (24)$$

Although the diagonalized metric in Eq. (22) is similar in form to that of the Fock-Lorentz case [2], it is not a Friedmann-Lobachevsky type metric because the spatial part of the metric, dl^2, is τ-dependent. Note also that the Planck-scale invariant t_P does not appear in the metric in Eqs. (19) and (21). Although the KMM transformation is singular at $t = t_P$ and, thus, the physics before and after this time must be treated separately, this implies that $t = t_P$ is not a geometrically special point. Although this fact appears to be unexpected, it is natural in view of the fact that the space-time structure of doubly special relativity is obtained by a nonlinear coordinate transformation, Eq. (9), from the Minkowski space-time; thus, the curvature tensor obtained from the metric in Eq. (11) vanishes.

We now consider the Fock-Lorentz(FL) transformation which is induced by the transformation, $V = e^{-\frac{t}{R}D}$:

$$\begin{aligned} t' &= \frac{\gamma(t - vx^3)}{1-(\gamma-1)\frac{t}{R} + \gamma v\frac{x^3}{R}}, && (25) \\ x'^3 &= \frac{\gamma(x^3 - vt)}{1-(\gamma-1)\frac{t}{R} + \gamma v\frac{x^3}{R}}, && (26) \\ x'^a &= \frac{x^a}{1-(\gamma-1)\frac{t}{R} + \gamma v\frac{x^3}{R}}, \quad a = 1, 2, && (27) \end{aligned}$$

where R is a large invariant length scale, and the direction of the relative motion between the two frames is assumed to be in the x^3-direction. Note that, if one defines new variable $\tilde{x}^\mu$ as

$$\tilde{x}^\mu = \frac{x^\mu}{1+\frac{t}{R}} \ , \tag{28}$$

this new variable transforms as a 4-vector under the linear Lorentz transformation, and the quantity $\tilde{t}^2 - \vec{\tilde{x}} \cdot \vec{\tilde{x}} = (t^2 - \vec{x} \cdot \vec{x})/(1+\frac{t}{R})^2$ can easily be shown to be invariant under the Fock-Lorentz transfomation [1-2]. By using Eq. (28), one finds the metric of the coordinate system,

$$ds^2 = \frac{1}{(1+\frac{t}{R})^4}[dt^2 - ((1+\frac{t}{R})d\vec{x} - \frac{\vec{x}}{R}dt)^2]. \tag{29}$$

This metric may be put into a diagonal form[2]:

$$ds^2 = \frac{R^4}{\hat{t}^4}(d\hat{t}^2 - \frac{\hat{t}^2}{R^2}dl^2), \tag{30}$$

where $t = \hat{t}\sqrt{1-\frac{r^2}{R^2}}$, and dl^2 is given by

$$dl^2 = \frac{1}{1-\frac{r^2}{R^2}}(\frac{dr^2}{1-\frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)). \tag{31}$$

The components of the metric tensor in Eq. (29) read

$$g_{00}^{FL} = \frac{1-\frac{\vec{x}\cdot\vec{x}}{R^2}}{(1+\frac{t}{R})^4}, \tag{32}$$

$$g_{0i}^{FL} = \frac{\frac{x^i}{R}}{(1+\frac{t}{R})^3}, \tag{33}$$

$$g_{ij}^{FL} = -\frac{\delta_{ij}}{(1+\frac{t}{R})^2}, \tag{34}$$

whose determinant is given by

$$g^{FL} = -(1+\frac{t}{R})^{-10}. \tag{35}$$

The maximum signal velocity in Fock-Lorentz nonlinear relativity is determined by the null geodesic condition, $ds^2 = 0$, in Eq. (29):

$$[(1+\frac{t}{R})\vec{v} - \frac{\vec{x}}{R}]^2 = 1. \tag{36}$$

Thus, in the direction where $\vec{x}$ is parallel to $\vec{v}$, the maximum signal velocity becomes

$$v = \frac{1}{1+\frac{t}{R}}(1+\frac{x}{R}). \tag{37}$$

As in the case of the KMM nonlinear relativity, the maximum signal velocity in the FL nonlinear relativity depends on the position in space-time. The velocity at the origin, $\vec{x}=0$, on the other hand, is maximum and has the value of the conventional light velocity at $t=0$ and monotonically decreases in time to become zero when t becomes infinite.

III. SOLUTIONS OF DOUBLY SPECIAL RELATIVISTIC MAXWELL'S EQUATIONS

To understand the physical implications of doubly special relativity, we solve Maxwell's equations invariant under the KMM transformation[11] and the Fock-Lorentz transformation[1]. Maxwell's equations, invariant under the nonlinear Lorentz transformations Eqs. (6)-(8) and Eqs. (25)-(27), are of the form

$$D_\alpha F^{\alpha\beta} = 4\pi J^\beta, \tag{38}$$

$$D_\alpha\, {}^*F^{\alpha\beta} = 0, \tag{39}$$

where D_α represents a covariant derivative induced by the metrics, Eq. (11) and Eq. (29), for the KMM and FL DSR, respectively. Thus, Eqs. (38) and (39) may be written as

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}F^{\alpha\beta})}{\partial x^\alpha} = 4\pi J^\beta, \tag{40}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\;{}^*F^{\alpha\beta})}{\partial x^\alpha} = 0. \tag{41}$$

To solve Eqs. (40) and (41), one finds it convenient to write $F^{\alpha\beta}$ in terms of the electric and magnetic fields, $E_i = F^{0i}, B_i = {}^*F^{0i}$. Then, the homogeneous equation, Eq. (41), becomes

$$\partial_i B_i = 0, \tag{42}$$

$$D_0 B_i + \epsilon_{ijk}\partial_j E_k = 0, \tag{43}$$

where $D_0 = \frac{1}{\sqrt{-g}}\partial_0\sqrt{-g}$, which reflects the fact that the space-time coordinates are nonlinear. These equations can be solved by writing the fields in terms of the potential A_μ;

$$B_i = \epsilon_{ijk}\partial_j A_k, \tag{44}$$

$$E_i = \partial_i A_0 - D_0 A_i. \tag{45}$$

The inhomogeneous equation, Eq. (40), becomes

$$\partial_i E_i = 4\pi J^0, \tag{46}$$

$$-D_0 E_i + \epsilon_{ijk}\partial_j B_k = 4\pi J_i. \tag{47}$$

Note that the system is invariant under the gauge transformation

$$A_i \to A_i' = A_i + \partial_i \Lambda, \tag{48}$$

$$A_0 \to A_0' = A_0 + D_0 \Lambda. \tag{49}$$

Substituting Eqs. (44) and (45) into Eqs. (46) and (47), one obtains the wave equation for A_μ:

$$(D_0^2 - \partial_i\partial_i)A_\mu = 4\pi J_\mu, \tag{50}$$

where we have used the gauge-fixing condition $-D_0 A_0 + \partial_i A_i = 0$. Note that Eq. (50) is equivalent to

$$(\partial_0^2 - \partial_i\partial_i)\tilde{A}_\mu = 4\pi \tilde{J}_\mu, \tag{51}$$

where

$$\tilde{A}_\mu = \sqrt{-g}A_\mu, \quad \tilde{J}\mu = \sqrt{-g}J_\mu. \tag{52}$$

One can now readily find the solutions of the doubly special relativistic Maxwell's equations. In the source-free region, the general solution is of the form

$$A^\mu(t,\vec{x}) = \frac{1}{\sqrt{-g}}\int d^3k d\omega \epsilon^\mu(\omega,\vec{k})e^{i\vec{k}\cdot\vec{x}-i\omega t}, \tag{53}$$

where ϵ^μ is the polarization vector satisfying

$$\epsilon^0\omega = \vec{\epsilon}\cdot\vec{k}, \tag{54}$$

and the dispersion relation is given by

$$\omega^2 - k^2 = 0. \tag{55}$$

This shows that the solutions of Maxwell's equations are just plane waves multiplied by an envelope function, $\frac{1}{\sqrt{-g}}$, coming from the background metric factor appearing in Eq. (52). Note that, for the KMM case, the determinant of the metric is

$$\sqrt{-g^{KMM}} = (1 - \frac{t_P}{t})^3, \tag{56}$$

and, for the Fock-Lorentz case, it is

$$\sqrt{-g^{FL}} = (1 + \frac{t}{R})^{-5}. \tag{57}$$

Thus, in the case of the KMM DSR, the amplitude of the wave decreases in time, starting from the Planck time when the amplitude is infinitely large. In the Fock-Lorentz case, on the other hand, the amplitude of the wave increases in time, starting from $t = 0$ when the amplitude is the same as it is in the conventional plane-wave case. It appears that energy is continuously extracted from (supplied to) the Maxwell system in the KMM case (FL case), although there is no source or sink in the system. Although, in the case of KMM, the Maxwell wave appears to have infinitely large energy near the Planck time, the wave velocity is not affected by the nonlinearity of the Lorentz group representation. This is also the case for the Fock-Lorentz relativity.

It is to be noted that the velocity of the Maxwell waves is the same as that of the conventional light wave, but is not the same as the maximum signal velocity, Eqs. (17) and (37) for the KMM and the FL relativity, respectively, which follow from the null geodesic condition. This is in marked contrast to the case of linear relativity, where light propagates along the null geodesics.

The fact that the energy appears to be supplied or extracted without any source or sink in DSR can also be seen in Poynting's theorem. From Maxwell's equations, Eqs. (46) and (47), one obtains the continuity equation

$$D_0 u + \frac{1}{4\pi}\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = -\vec{J} \cdot \vec{E}, \tag{58}$$

where

$$u = \frac{1}{8\pi} \int d^3x (E^2 + B^2). \tag{59}$$

Note that the first term in Eq. (58) can be written as

$$D_0 u = \partial_0 u + \frac{1}{\sqrt{-g}} (\partial_0 \sqrt{-g})\; u. \tag{60}$$

The second term of Eq. (60) may act like a source term, and due to this term, the energy-momentum of the wave is not conserved globally.

IV. DISCUSSION

DSR in momentum space is obtained by defining the modified algebra [4,5,8,11]

$$K^i = U^{-1} {L_0}^i \, U, \tag{61}$$

where $L_{\alpha\beta} = p_\alpha \frac{\partial}{\partial p^\beta} - p_\beta \frac{\partial}{\partial p^\alpha}$ are the generators of the linear Lorentz transformation. From the modified generators, K^i, one obtains the modified Lorentz transformation in momentum space, which in turn gives the modified dispersion relation

$$E^2 f^2(E) - p^2 g^2(E) = m^2, \tag{62}$$

where the forms of the functions f and g are determined by the form of the transformation U. From the dispersion relation in Eq. (62), one defines the group velocity of the particle in question and discusses the effects in Planck-scale physics [7].

Although p in the linear Lorentz generators, $L_{\alpha\beta}$, is clearly the momentum of the particle, it is not clear what represents the variable p appearing in the modified generators, K^i, and in the nonlinear Lorentz transformation. Thus, it is not clear whether the "group velocity" defined from the dispersion relation in Eq. (62) represents the true group velocity of the particle in question. We believe that this fact is the source of the questions raised by the authors of Refs. [9,10].

One way to define the velocity of a particle unambiguously is to solve the field equation corresponding to the particle in question. Since the exact nonlinear Lorentz transformation in position space corresponding to the momentum-space DSR relevant in Planck-scale physics [4,5] is not yet known, we solved Maxwell's equations invariant under the KMM transformation[11] and the Fock-Lorentz transformation[1] in an attempt to understand how the light wave propagates in DSR. As we showed in the last section, although the Maxwell wave in KMM nonlinear relativity appears to acquire infinitely large energy near the Planck time, its velocity is the same as that of conventional light waves. The reason is that the nonlinear Lorentz transformation does not affect the phase of the Maxwell wave, but contributes only to the amplitude of the wave.

Another puzzle present in doubly special relativity defined in position space is that the velocity of the doubly special relativistic Maxwell wave is not the same as the maximum signal velocity determined by the null geodesics. The maximum signal velocity in KMM

nonlinear relativity is infinitely large near the Planck time and approaches the conventional light velocity as time becomes large, which is exactly the property that is needed for application to Planck-scale physics. The velocity of the solution of Maxwell's equations invariant under the KMM transformation, however, is exactly the same as the conventional light velocity, although its energy becomes infinitely large near the Planck time. This raises many interesting questions, such as the one concerning the relation between null geodesics and the particle trajectory or wave propagation, whether there exists a particle or a wave that travels with the maximum signal velocity determined by the null geodesics, *etc.* This also raises another serious question on the causality of the theories based on the KMM transformations because the light cone determined by Maxwell's equations may be inside or outside the null cone determined by the null geodesic equation, Eq. (16), depending on the position of the particle in question. Note that the dispersion relation, Eq. (62), is obtained from the metric of the coordinate system defined in momentum space. However, our analysis indicates that the signal velocity determined by the metric may not be the same as that determined by the solutions of the field equations. Thus, if the physics of doubly special relativity are to be understood more clearly, further investigations of the field equations invariant under the nonlinear Lorentz transformations and the implications of the null geodesics are needed.

Acknowledgments

This work was supported in part by the Sabbatical Year Program of Ewha Woman's University (SKK), by the Yonsei University Research Fund (SMK), by the Basic Research Program of the Korea Science and Engineering Foundation under Grant number R01-1999-000-00018-0(2003)(CR), and by Korea Research Foundation under Project number KRF-2003-005-C00010 (JHY).

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